(Pages : 4)

D – 3116

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, December 2017
(First Degree Programme Under CBCSS)
MATHEMATICS
Core Course – 2
MM 1341 : Algebra and Calculus – 1
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Is the set $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ a ring with respect to addition modulo 4 and multiplication modulo 4 ?
2. Give an example of a ring which has no zero divisors.
3. Find $[2]_5.[3]_5$.
4. Define exponent of a group.
5. Find the distance from the point $(-7, 5, 2)$ to the xy-plane.
6. Fill in the blanks :
   In 3-space, the equation $x^2 + z^2 = 10$ represents a _____.
7. Find the cosine of the angle between the vectors $\vec{u} = \langle 0, 2, 3 \rangle$ and $\vec{v} = \langle 0, -2, 1 \rangle$.
8. Give the natural domain for the vector valued function.
   $\vec{r}(t) = (\ln|t-6|)\,\vec{i} + 7t^2e^t\,\vec{j} + \sqrt{t-2k}$
9. Find the slope of the line in 2 space that is represented by the vector equation $\vec{r} = (12-7t)\,\vec{i} - (5-4t)\,\vec{j}$.
10. State Kepler's Law of Areas.

(10×1=10 Marks)

P.T.O.

All Saints' College Trivandrum

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Justify that the set $\mathbb{Z} - \{3\}$ of all integers except 3 is not a ring with respect to the usual addition and multiplication.

12. Prove that in $\mathbb{Z}/m\mathbb{Z}$, $[a]_m$ is a unit if and only if $(a, m) = 1$.

13. If R and S are rings and the function $f : R \to S$ is a homomorphism, then prove that $f(-r) = -f(r)$ for any r in R.

14. Find the unit vector in the same direction from the point A(–3, 2) to the point B(1, –1).

15. If $\vec{u}$ and $\vec{v}$ are vectors in 3-space, then prove that $\vec{v}.\left(\vec{u}\times\vec{v}\right)=0$.

16. Find $\vec{u}.\left(\vec{v}\times\vec{w}\right)$ where $\vec{u}=2\vec{i}-3\vec{j}+\vec{k}$, $\vec{v}=4\vec{i}+\vec{j}-3\vec{k}$, $\vec{w}=\vec{j}+5\vec{k}$.

17. Find parametric equations for the line through (–5, 2) that is parallel to $2i - 3j$.

18. Find a spherical coordinate equation of the Cartesian equation $z=\sqrt{x^2+y^2}$.

19. Convert the following spherical coordinates to rectangular coordinates :

    a) $(7, 0, \pi/2)$ b) $(1, \pi, 0)$

20. If $\vec{r}(t)$ is a vector-valued function in two or three dimensional space and $\left\|\vec{r}(t)\right\|$ is constant for all t, then prove that $\vec{r}(t)$ and $\vec{r}'(t)$ are orthogonal vectors for all t.

21. Find a change of parameter $t = g(\tau)$ for the semicircle

$$\vec{r}(t)=\cos t\,\vec{i}+\sin t\,\vec{j},\ 0\le t\le\pi$$

such that the semicircle is traced counter clockwise as τ increases over the interval [0, 1].

22. Find the curvature for the helix

$$\vec{r}(t)=(a\cos t)\,\vec{i}+(a\sin t)\,\vec{j}+bt\,\vec{k}\quad a, b \ge 0,\ a^2+b^2\neq 0.$$

(8×2=16 Marks)

All Saints' College Trivandrum

Answer **any 6** questions from the questions **23** to **31**. These questions carry **4** marks **each**.

23. Let R be a commutative ring with atleast 5 elements. Prove that the equation $x^2 - rx + s = 0$ has at most two solutions in R for every r, s in R, if and only if R has no zero divisors.

24. Define field. Give one example.

25. Find the coordinates of the centre and the radius of the sphere
$x^2 + y^2 + z^2 - 4x + 2y - 2z - 10 = 0$.

26. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors in 3-space, then prove that
$\vec{u} \times \left(\vec{v} + \vec{w}\right) = \left(\vec{u} \times \vec{v}\right) + \left(\vec{u} \times \vec{w}\right)$.

27. Determine whether the following lines are parallel, intersect or are skew. If they intersect, find the point of intersection.
(Line 1) : $x = 3 + 2t$, $y = 2 + t$, $z = -2 + 2t$, $-\infty < t < \infty$
(Line 2) : $x = 1 + 4s$, $y = 1 + 2s$, $z = -3 + 4s$, $-\infty < s < \infty$

28. Determine whether the following vector-valued function has continuously turning tangent vector :
$\vec{r}(t) = 3t^2\,\vec{i} + (t^2 - 2t)\,\vec{j} + t^3\,\vec{k}$

All Saints College Trivandrum

29. Find the unit tangent vector at a point t to the curve $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j}$.

30. Suppose that a particle moves through 3-space so that its position vector at time t is $\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + t^3\,\vec{k}$.
Find
a) the scalar tangential and normal components of acceleration at time t = 1.
b) the curvature of the path at the point where the particle is located at t = 1.

31. A shell is fired from ground level with a muzzle speed of 320 ft/s and elevation angle of 60°. Find the horizontal distance travelled by the shell. **(6×4=24 Mark**

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) If R is any commutative ring, prove that the set of n-element column vectors for fixed n is an abelian group under addition.

b) Let g_0 be the exponent of a finite abelian group G. Then the order of any element b of G divides g_0.

33. a) State and prove the Chinese Remainder Theorem.

b) Find a solution of the following simultaneous congruences :

$x \equiv 3 \pmod{11}$,

$x \equiv 6 \pmod{8}$,

$x \equiv -1 \pmod{15}$

Also find the smallest non-negative solution.

34. a) Find the point where the line

$$x = \frac{8}{3} + 2t,\ y = -2t,\ z = 1 + t$$

intersects the plane $3x + 2y + 6z = 6$.

b) Identify the surface $x^2 - y + z^2 = 0$.

35. a) Find the unit tangent vector to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at the point $t = 1$.

b) Suppose that a particle moves along a curve so that its position vector at time t is $\left(\vec{r} = e^t \vec{i} + e^{-t} \vec{j} + \sqrt{2}t \vec{k}\right)$.

Find the distance traveled and the displacement of the particle during the time interval $0 \leq t \leq \ln 3$.

(2×15=30 Ma

All Saints' College Trivandrum